Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference
9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

a) Find $\frac{dy}{dx}$ by bringing the power down and subtracting 1 from the power

$$y = 3x^4 - 8x^3 - 3$$

← the three has no power of x so it disappears.

$$\frac{dy}{dx} = 4(3x^3) - 3(8x^2)$$

$$\boxed{\frac{dy}{dx} = 12x^3 - 24x^2}$$

To find $\frac{d^2y}{dx^2}$, differentiate again.

$$\frac{dy}{dx} = 12x^3 - 24x^2$$

$$\frac{d^2y}{dx^2} = 3(12x^2) - 2(24x^1)$$

$$\boxed{\frac{d^2y}{dx^2} = 36x^2 - 48x}$$

b) Sub $x=2$ into $\frac{dy}{dx}$. If it equals 0, the gradient is 0 so there is a stationary point

$$\frac{dy}{dx} = 12x^3 - 24x^2$$

@ $x=2$ $\quad \frac{dy}{dx} = 12(2)^3 - 24(2)^2$

$$= (12 \times 8) - (24 \times 4)$$

Question 1 continued

$\boxed{@x=2 \quad \frac{dy}{dx}=0 \text{ , so there is a stationary point at } x=2}$

c) when $\frac{d^2y}{dx^2} > 0 \rightarrow$ point is a minimum

$\frac{d^2y}{dx^2} < 0 \rightarrow$ point is a maximum

$$\frac{d^2y}{dx^2} = 36x^2 - 48x$$

Sub in $x=2$ to find $\frac{d^2y}{dx^2}$ when $x=2$ $\rightarrow$ $@x=2$

$$\frac{d^2y}{dx^2} = 36(2)^2 - 48(2)$$

$$= 48$$

$\boxed{\text{At } x=2,\ \frac{d^2y}{dx^2}=48 \quad 48>0 \text{, so the point is a minimum}}$

(Total for Question 1 is 7 marks)

2.

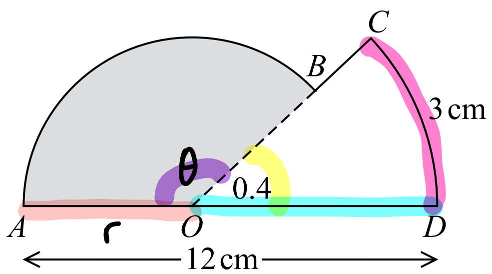


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, ∠*COD* = 0.4 radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

a) $\boxed{\text{arc length} = r\theta}$

arc length $= r\theta$

OD = R

$r = \dfrac{\text{arc length}}{\theta}$

$r = \dfrac{3}{0.4}$

$r = 7.5$

$\boxed{OD = 7.5}$

b) $\boxed{\text{sector area} = \tfrac{1}{2}r^2\theta}$

$\theta = \pi - 0.4$

$r = AD - OD$

$= 12 - 7.5$

$r = 4.5$

sector area $= \tfrac{1}{2}r^2\theta$

$= \tfrac{1}{2} \times 4.5^2 \times (\pi - 0.4)$

$= 10.125(\pi - 0.4)$

$= \boxed{27.8\text{cm}^2}$

(Total for Question 2 is 5 marks)

3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

a) Complete the square to find an equation for C

$$x^2 + y^2 - 4x + 10y = k$$

$$(x-2)^2 - 4 + (y+5)^2 - 25 = k$$

$$(x-2)^2 + (y+5)^2 = k + 29$$

In the form $(x-a)^2 + (y-b)^2 = r^2$, (a, b) is the centre

$$\boxed{\text{centre} = (2, -5)}$$

b) The radius of a circle (r) must be positive, so r^2 must also be positive

$$r^2 = k + 29$$

$$k + 29 > 0$$

$$k > -29$$

$$\boxed{k > -29}$$

(Total for Question 3 is 4 marks)

4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

$$\int_a^{2a} \frac{t+1}{t}\,dt$$

$\frac{t+1}{t} = \frac{t}{t} + \frac{1}{t} = 1 + \frac{1}{t}$

$$= \int_a^{2a} 1 + \frac{1}{t}\,dt$$

integrate with respect to t
$\int 1 = t$ because you add 1 to the power to make the power of t 1.

$$\boxed{\int \frac{1}{x} = \ln x}$$

$$= \left[t + \ln t\right]_a^{2a}$$

Sub in limits

$$= (2a + \ln 2a) - (a + \ln a)$$

$$= 2a + \ln 2a - a - \ln a$$

$$= a + \ln 2a - \ln a$$

$$= a + \ln 2$$

$$\boxed{\begin{aligned} &\ln x - \ln y = \ln \tfrac{x}{y} \\ &\therefore \ln 2a - \ln a = \ln \tfrac{2a}{a} \\ &\qquad = \ln 2 \end{aligned}}$$

rearrange for a →

$$a + \ln 2 = \ln 7$$

← set equal to ln 7 given in the question

$$a = \ln 7 - \ln 2$$

$$\boxed{\text{Use same rule: } \ln x - \ln y = \ln \tfrac{x}{y}}$$

$$a = \ln \frac{7}{2}$$

$$\boxed{k = \frac{7}{2}}$$

(Total for Question 4 is 4 marks)

5. A curve C has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where a and b are integers to be found.

(3)

Rearrange $x = 2t - 1$ to make the subject t, then sub into $y = 4t - 7 + \frac{3}{t}$

$$x = 2t - 1$$
$$x + 1 = 2t$$
$$t = \frac{x+1}{2}$$

$$y = 4t - 7 + \frac{3}{t}$$
$$= 4\left(\frac{x+1}{2}\right) - 7 + \frac{3}{\left(\frac{x+1}{2}\right)}$$
$$= 2(x+1) - 7 + \frac{6}{x+1}$$

multiply numerator and denominator of $2(x+1)$ and 7 by $(x+1)$ to give all terms a common denominator of $(x+1)$

$$= \frac{2(x+1)^2}{x+1} - \frac{7(x+1)}{x+1} + \frac{6}{x+1}$$

add all terms together because they have a common denominator

$$= \frac{2(x+1)^2 - 7(x+1) + 6}{x+1}$$

simplify

$$= \frac{2x^2 + 4x + 2 - 7x - 7 + 6}{x+1}$$

$$\boxed{= \frac{2x^2 - 3x + 1}{x+1}}$$

(Total for Question 5 is 3 marks)

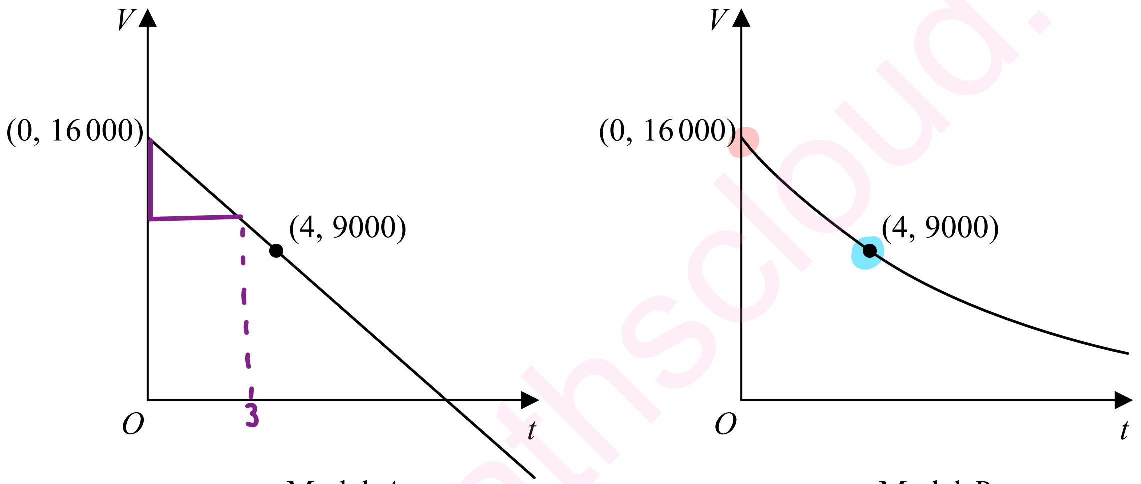
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Model A Model B

(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

a i) $16{,}000 - \frac{3}{4}(16000 - 9000)$ ← we want to find 3/4 of the difference between 16000 and 9000 and subtract it from 16,000

$= 16{,}000 - \frac{3}{4}(7000)$

$= \boxed{10{,}750 \text{ barrels}}$

ii) the model predicts that the daily volume of oil would eventually become negative, which is impossible.

Question 6 continued

b i) Use the model $V = Ae^{kt}$, we need to find A and k by subbing in the two points on the graph.

$$V = Ae^{kt}$$

@ $t = 0$
$V = 16{,}000$

$$V = Ae^{kt}$$
$$16{,}000 = Ae^{0t} \quad e^{0} = 1$$
$$16{,}000 = A$$

@ $t = 4$
$V = 9000$
$A = 16{,}000$

$$V = Ae^{kt}$$
$$9000 = 16{,}000e^{4k}$$
$$\tfrac{9}{16} = e^{4k}$$
$$\ln \tfrac{9}{16} = 4k$$
$$k = \tfrac{1}{4}\ln \tfrac{9}{16}$$
$$k = -0.144$$

$$V = Ae^{kt} \rightarrow \boxed{V = 16{,}000e^{-0.144t}}$$

ii) sub in $t = 3$ to find V 3 years after the start of drilling

$$V = 16{,}000e^{-0.144t}$$

@ $t = 3$
$$V = 16{,}000e^{-0.144 \times 3}$$
$$= 16{,}000e^{-0.432}$$
$$= 10{,}400 \ (3sf)$$

$$\boxed{10{,}400 \text{ barrels}}$$

(Total for Question 6 is 7 marks)

7.

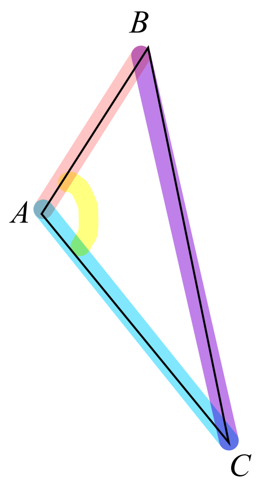


Figure 2

Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

Find $\overrightarrow{AC}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2i + 3j + k + i - 9j + 3k$ ↓ add i, j, k components separately

$\overrightarrow{AC} = 3i - 6j + 4k$

Now find the length of all three sides using 3d pythagoras

$|AC| = \sqrt{3^2 + (-6)^2 + 4^2}$ magnitude $= \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{61}$

$|BC| = \sqrt{1^2 + (-9)^2 + 3^2}$

$= \sqrt{91}$

$|AB| = \sqrt{2^2 + 3^2 + 1^2}$

$= \sqrt{14}$

DO NOT WRITE IN THIS AREA

Question 7 continued

Use formula $a^2 = b^2 + c^2 - 2bc\cos A$

rearrange to find $\cos A$ → $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ where $a = |BC| = \sqrt{91}$

$A = \angle BAC$ $\qquad b = |AC| = \sqrt{61}$

$c = |AB| = \sqrt{14}$

$$\cos BAC = \frac{61 + 14 - 91}{2 \times \sqrt{61} \times \sqrt{14}}$$

$$\cos BAC = \frac{-16}{58.45}$$

$$= -0.274$$

$$\boxed{\angle BAC = 105.9}$$

(Total for Question 7 is 5 marks)

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

a) Find f(3.5) and f(4). If there is a sign change between them, (one is negative and one is positive), the root lies in that interval

$$f(3.5) = \ln(2(3.5) - 5) + 2(3.5)^2 - 30$$
$$= -4.8$$

$$f(4) = \ln(2(4) - 5) + 2(4)^2 - 30$$
$$= 3.1$$

The function f(x) is continuous in the interval [3.5, 4] and there is a change of sign between f(3.5) and f(4), so there is a root between x = 3.5 and x = 4

b) Use the Newton Raphson formula in the formula booklet

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 4 - \frac{3.099}{16.67}$$

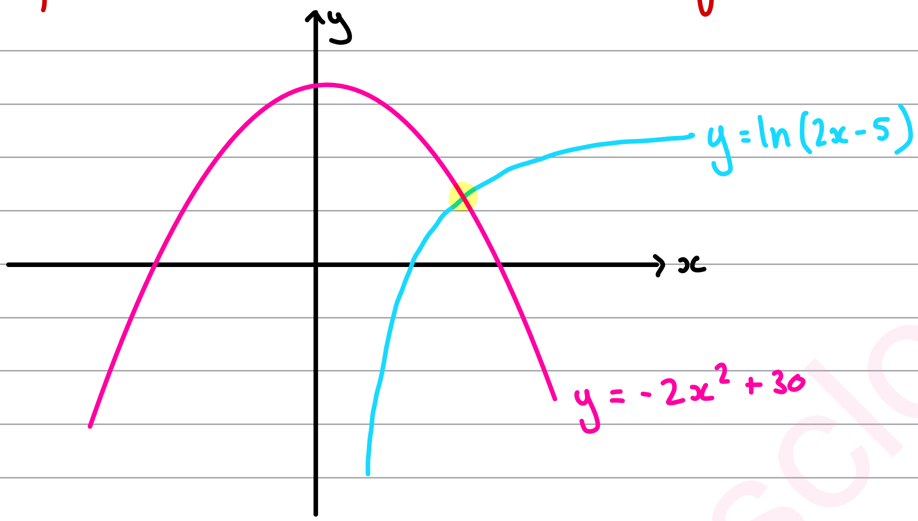
$$\boxed{x_1 = 3.81}$$

Question 8 continued

c) $f(x) = 0 \rightarrow \ln(2x-5) + 2x^2 - 30 = 0$

$\ln(2x-5) = -2x^2 + 30$

Sketch $y = \ln(2x-5)$ and $y = -2x^2 + 30$ on a graph. The number of intersections between them is equal to the number of roots.



There is 1 point of intersection, so α is the only root

(Total for Question 8 is 6 marks)

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

use rule $\tan\theta = \frac{\sin\theta}{\cos\theta}$

and $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$$\tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

multiply top and bottom by $\sin\theta$ and $\cos\theta$ respectively so both fractions have the same denominator

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

use rule: $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

use rule $2\sin\theta\cos\theta = \sin 2\theta$

$\therefore \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$$= \frac{1}{\frac{1}{2}\sin 2\theta}$$

$\frac{1}{\sin x} = \operatorname{cosec} x$

$\therefore \frac{2}{\sin 2\theta} = 2\operatorname{cosec}2\theta$

$$= \frac{2}{\sin 2\theta}$$

$$= 2\operatorname{cosec}2\theta$$

$$\therefore \tan\theta + \cot\theta = 2\operatorname{cosec}2\theta$$

b) Because $\tan\theta + \cot\theta = 2\operatorname{cosec}2\theta$, the equation can be written as $2\operatorname{cosec}2\theta = 1$

$$\frac{2}{\sin 2\theta} = 1$$

$$\sin 2\theta = 2$$

the maximum value of $\sin 2\theta$ is 1, and $2 > 1$, so there are no solutions.

(Total for Question 9 is 5 marks)

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10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ← In formula booklet

$f(x) = f(\theta) = \sin\theta$
$f(x+h) = f(\theta+h) = \sin(\theta+h)$

use compound angle formula:
$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$$f'(\theta) = \lim_{h \to 0} \frac{\sin(\theta+h) - \sin\theta}{h}$$

$$= \lim_{h \to 0} \frac{\sin\theta\cos(h) + \sin(h)\cos\theta - \sin\theta}{h}$$

factorise out $\sin\theta$ and separate into 2 fractions

$$= \lim_{h \to 0} \frac{\sin\theta(\cos(h) - 1)}{h} + \frac{\sin(h)\cos\theta}{h}$$

as $h \to 0$
$\frac{\sinh}{h} \to 1$
$\frac{\cosh - 1}{h} \to 0$

as $h \to 0$ $f'(\theta) = \sin\theta \times 0 + 1 \times \cos\theta$

$= \cos\theta$

$$\boxed{f'(\theta) = \cos\theta}$$

(Total for Question 10 is 5 marks)

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.
(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.
(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.
(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.
(2)

a) We can find the horizontal distance travelled by setting the height (H) to 0, and solving for d.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = -0.002$

$b = 0.4$

$c = 1.8$

$$H = 1.8 + 0.4d - 0.002d^2$$

$$0 = 1.8 + 0.4d - 0.002d^2$$

$$d = \frac{-0.4 \pm \sqrt{0.4^2 - (4 \times -0.002 \times 1.8)}}{2 \times -0.002}$$

$$d = \cancel{-4.403}, \quad 204.43$$

d must be positive so not valid

$\boxed{d = 204\text{m } (3sf)}$

Question 11 continued

b) @$d=0 \rightarrow H = 1.8 + 0.4(0) - 0.002(0^2)$
$= 1.8$

$\boxed{\text{1.8m is the initial height of the arrow above the ground}}$

c) Complete the square to get the equation in the required form

$1.8 + 0.4d - 0.002d^2$ factorise out -0.002

$= -0.002\left[d - 200d\right] + 1.8$ complete the square of $(d-200)$

$= -0.002\left[(d-100)^2 - 10{,}000\right] + 1.8$

$= -0.002(d-100)^2 + 20 + 1.8$

$= -0.002(d-100)^2 + 21.8$

$\boxed{= 21.8 - 0.002(d-100)^2}$

d) i) Find the maximum point from the first equation. In the form $c + a(d-b)^2$, the maximum point is $(-b, c)$

maximum point $= (100, 21.8)$
$\therefore$ maximum height $= 21.8\text{m}$

The adapted formula is the original formula + 0.3.

maximum height $= 21.8 + 0.3$
$= \boxed{22.1\text{m}}$

ii) Set H to 22.1 and solve to find d

(Total for Question 11 is 9 marks)

Question 11 continued

Use quadratic formula

$$\boxed{d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$22.1 = -0.002d^2 + 0.4d + 2.1$$

$$0 = -0.002d^2 + 0.4d - 20$$

$$d = \frac{-0.4 \pm \sqrt{0.4^2 - (4 \times -0.002 \times -20)}}{2 \times -0.002}$$

$$\boxed{d = 100\text{ m}}$$

(Total for Question 11 is 9 marks)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

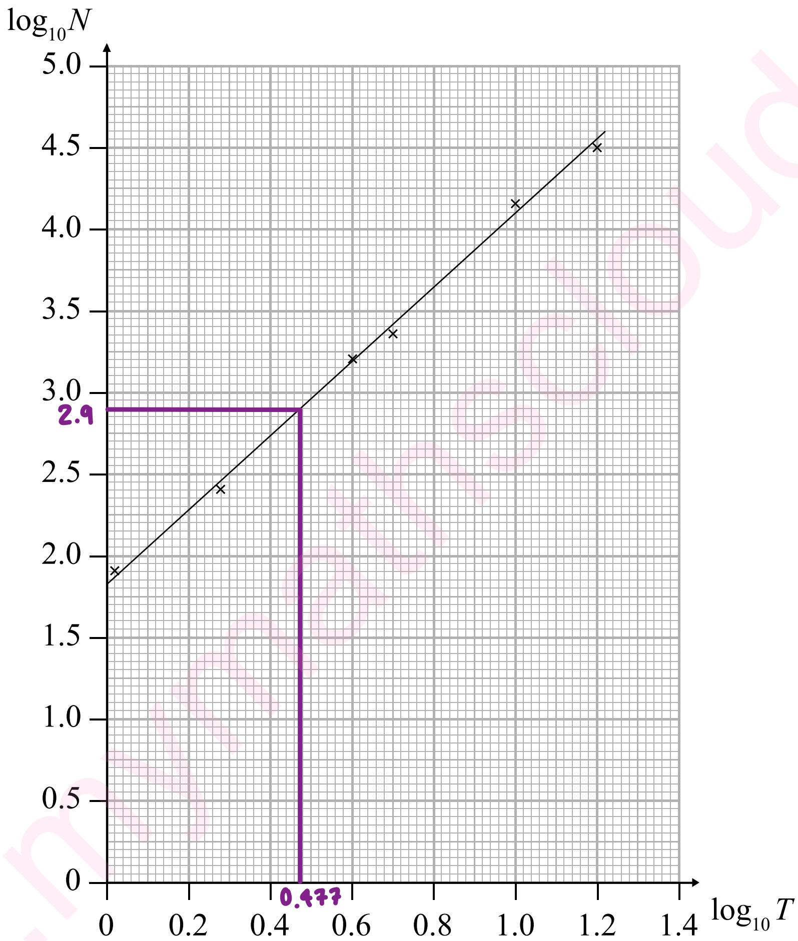

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

Question 12 continued

a) $N = aT^b$

$\log xy = \log x + \log y$

$\therefore \log aT^b = \log a + \log T^b$

take logs of both sides

$\log N = \log aT^b$

$\log N = \log a + \log T^b$

$\log_x a^b = b\log_x a$

$\log N = \log a + b\log T$

$\boxed{\log N = b\log T + \log a}$

b) Find $\log_{10} T$ when $T = 3$

$\log_{10} 3 = 0.477$

Find $\log_{10} T = 0.477$ on the graph, and find its corresponding $\log_{10} N$ value

$\log_{10} T = 0.477$

$\log_{10} N = 2.9$

$\log_{10} N = 2.9$ ← Find N

$N = 10^{2.9}$

$= 794$

$\boxed{N = 800 \quad (2sf)}$

c) When $N = 1{,}000{,}000$, $\log_{10} N = 6$

$\log_{10} N = 6$ is greater than the range of values on the graph, and we cannot extrapolate the graph and assume the model still holds.

d) $N = aT^b$

When $T = 1$, $N = a$, so a is the number of microbes 1 day after the start of the experiment

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

a) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{dy/dt}{dx/dt}$

$y = \sqrt{3}\cos 2t$

$\frac{dy}{dt} = -2\sqrt{3}\sin 2t$

$f(x) = \cos kx$
$f'(x) = -k\sin kx$

$x = 2\cos t$

$\frac{dx}{dt} = -2\sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{-2\sqrt{3}\sin 2t}{-2\sin t}$

$= \boxed{\frac{\sqrt{3}\sin 2t}{\sin t}}$

b) Find the gradient of the tangent of C at P

@ $t = \frac{2\pi}{3}$ $\quad \frac{dy}{dx} = \frac{\sqrt{3}\sin\left(2 \times \frac{2\pi}{3}\right)}{\sin\left(2\pi/3\right)} = -\sqrt{3}$

Question 13 continued

gradient of the normal $= \frac{1}{\sqrt{3}}$ (because gradients of perpendicular lines multiply to give -1 : $-\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$)

Find gradient of P by subbing $t = \frac{2\pi}{3}$ into equations of x and y

$$x = 2\cos t$$
$$= 2\cos\left(\tfrac{2\pi}{3}\right)$$
$$= -1$$
$$y = \sqrt{3}\cos(2t)$$
$$= \sqrt{3}\cos\left(\tfrac{4\pi}{3}\right)$$
$$= -\sqrt{3}/2$$

$$P = \left(-1, -\sqrt{3}/2\right)$$

Sub in $P = (-1, -\sqrt{3}/2)$ and gradient $= \frac{1}{\sqrt{3}}$ into $y = mx + c$ to find the equation

$m = \frac{1}{\sqrt{3}}$

$@\ x = -1\ y = -\sqrt{3}/2$

$$y = mx + c$$
$$y = \tfrac{1}{\sqrt{3}}x + c$$
$$-\sqrt{3}/2 = \tfrac{1}{\sqrt{3}} \times -1 + c$$
$$c = -\sqrt{3}/6$$

$$y = \tfrac{1}{\sqrt{3}}x - \sqrt{3}/6$$

multiply each term by $2\sqrt{3}$ to get the equation into the required form

$$2\sqrt{3}y = 2x - 1$$

$$\boxed{0 = 2x - 2\sqrt{3}y - 1}$$

c) Sub $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to find an in terms of $\cos(t)$.

$$2x - 2\sqrt{3}y - 1 = 0$$

$x = 2\cos t$, $y = \sqrt{3}\cos 2t \rightarrow$ $2(2\cos t) - 2\sqrt{3}(\sqrt{3}\cos 2t) - 1 = 0$

to find a quadratic equation $\rightarrow$ $\boxed{\begin{aligned}\cos 2t &= \cos^2 t - \sin^2 t \\ &= 2\cos^2 t - 1\end{aligned}}$

$$4\cos t - 6\cos 2t - 1 = 0$$
$$4\cos t - 6(2\cos^2 t - 1) - 1 = 0$$
$$4\cos t - 12\cos^2 t + 6 - 1 = 0$$

Question 13 continued

multiply by -1

$$4\cos t - 12\cos^2 t + 5 = 0$$

$$12\cos^2 t - 4\cos t - 5 = 0$$

$$(6\cos t - 5)(2\cos t + 1) = 0$$

$6\cos t = 5$ $\quad$ $2\cos t = -1$

$\cos t = 5/6$ $\quad$ $\cos t = -1/2$

$t = 0.586$ $\quad$ $\cancel{t = 2\pi/3}$

$t = 2\pi/3$ is at the point P which we have already found

Sub into equations of x and y to find the co-ordinates

$$x = 2\cos t = 2(5/6) = 5/3$$

$$y = \sqrt{3}\cos 2t = \sqrt{3}(2\cos^2 t - 1) = \sqrt{3}(2(5/6)^2 - 1) = \frac{7\sqrt{3}}{18}$$

use $\cos 2t = 2\cos^2 t - 1$ to find the y co-ordinate

$$\boxed{\text{co-ordinates : } \left(5/3,\ \frac{7\sqrt{3}}{18}\right)}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 13 continued

(Total for Question 13 is 13 marks)

14.

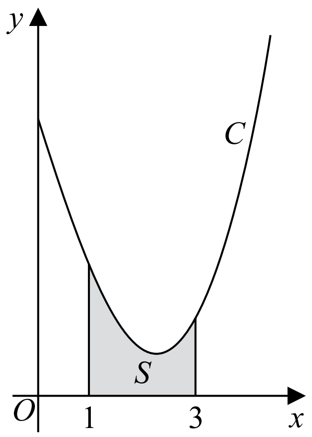

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

a) Use formula in formula booklet :

$$\boxed{\int_a^b y\,dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}}$$

$h = \frac{3-1}{4} = \frac{1}{2}$

$S = \frac{1}{2} \times \frac{1}{2} \{3 + 2.2958 + 2(2.3041 + 1.9242 + 1.9089)\}$

$= \frac{1}{4} (5.2958 + 2(6.1372))$

$= \boxed{4.393}$

Question 14 continued

b) Use more trapezia by increasing the width of the strips

c) $y = \frac{x^2 \ln x}{3} - 2x + 5$

$$\int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \; dx$$

$$\frac{1}{3}\int_1^3 \boxed{x^2 \ln x} \; \boxed{-6x + 15} \; dx$$

Integrate $x^2\ln x$ by parts using the formula

$$\boxed{\int uv' \, dx = uv - \int vu' \, dx}$$

$\int x^2 \ln x$

$u = \ln x \qquad v = \frac{1}{3}x^3$

$u' = \frac{1}{x} \qquad v' = x^2$

$= uv - \int vu'$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x}$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$

Integrate $-6x + 15$ by adding 1 to the power and dividing by the new power

Sub in limits

$= \frac{1}{3}\left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 - 3x^2 + 15x\right]_1^3$

$= \frac{1}{3}\left(\left(\frac{1}{3}(3)^3 \ln(3) - \frac{1}{9}(3^3) - 3(3^2) + 15(3)\right) - \left(\frac{1}{3}(1^3)\ln 1 - \frac{1}{9}(1^3) - 3(1^2) + 15(1)\right)\right)$

$\ln 1 = 0$

$= \frac{1}{3}\left((9\ln 3 - 3 - 27 + 45) - (0 - \frac{1}{9} - 3 + 15)\right)$

$= \frac{1}{3}\left(9\ln 3 + \frac{28}{9}\right)$

$= 3\ln 3 + \frac{28}{27}$

$= \ln 3^3 + \frac{28}{27}$

$= \ln 27 + \frac{28}{27}$

$\boxed{\text{Use rule: } a\ln b = \ln b^a}$

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15.

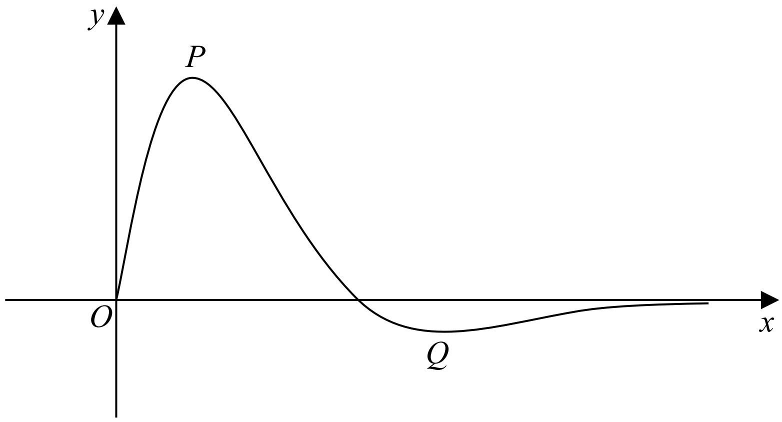

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

when gradient = 0

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

a) At the turning points, the gradient = 0

Use quotient rule:

$$\boxed{f(x) = \frac{g(x)}{h(x)} \qquad f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}}$$

$g(x) = 4\sin 2x \qquad g'(x) = 8\cos 2x$

$\frac{d}{dx}(\sin kx) = k\cos kx$

$h(x) = e^{\sqrt{2}x - 1} \qquad h'(x) = \sqrt{2}e^{\sqrt{2}x - 1}$

$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

Question 15 continued

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}$$

use the quotient rule to differentiate

$$f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}$$

$$f'(x) = \frac{8\cos 2x \times e^{\sqrt{2}x-1} - \sqrt{2}e^{\sqrt{2}x-1} \times 4\sin 2x}{(e^{\sqrt{2}x-1})^2}$$

factorise out $e^{\sqrt{2}x-1}$

$$= \frac{e^{\sqrt{2}x-1}(8\cos 2x - 4\sqrt{2}\sin 2x)}{e^{\sqrt{2}x-1} \times e^{\sqrt{2}x-1}}$$

we are finding the turning points, so set the gradient equal to 0

$$= \frac{8\cos 2x - 4\sqrt{2}\sin 2x}{e^{\sqrt{2}x-1}} = 0$$

multiply each side by $e^{\sqrt{2}x-1}$

$$8\cos 2x - 4\sqrt{2}\sin 2x = 0$$

$$8\cos 2x = 4\sqrt{2}\sin 2x$$

$$1 = \frac{4\sqrt{2}\sin 2x}{8\cos 2x}$$

$\frac{\sin kx}{\cos kx} = \tan kx$

$$1 = \frac{\sqrt{2}}{2}\tan 2x$$

$$\tan 2x = \frac{2}{\sqrt{2}}$$

$$\boxed{\tan 2x = \sqrt{2}}$$

b) $f(x) \longrightarrow$ turning point at $\tan 2x = \sqrt{2}$
$f(2x) \longrightarrow$ turning point at $\tan 4x = \sqrt{2}$

we are finding the minimum turning point. The first solution would give the max turning point (see graph). So add on π (because the period of $\tan x$ is π).

$$\tan 4x = \sqrt{2}$$

$$x = \frac{\tan^{-1}\sqrt{2} + \pi}{4}$$

$$\boxed{= 1.02}$$

Question 15 continued

$f(x) \longrightarrow$ turning point at $\tan 2x = \sqrt{2}$

$3 - 2f(x) \rightarrow$ turning point at $\tan 2x = \sqrt{2}$

it is the same because the 3 and -2 both translate the graph vertically, so the x co-ordinates stay the same

$\tan 2x = \sqrt{2}$

$x = \dfrac{\tan^{-1}\sqrt{2}}{2}$

$\boxed{x = 0.478}$

we are finding the first solution here because in the translation $-f(x)$, the graph is reflected in the x axis, the the maximum point becomes the minimum and vice versa.

(Total for Question 15 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS